ELECTROMAGNETIC SCATTERING FROM A MULTI-LAYERED SURFACE WITH LOSSY INHOMOGENEOUS DIELECTRIC PROFILES FOR REMOTE SENSING OF SNOW

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Abstract—A multilayered backscattering model for a lossy medium has been presented in this paper. This multilayered model has been used to calculate the total surface reflection coefficients of a snow pack for both horizontal and vertical co-polarizations. The total surface reflection coefficients include contributions from both surface and volumetric backscattering. The backscattering coefficients calculated by this model were compared with in situ measurements on dry and wet snow. Results show that good agreements are obtained between the model and measurements for the co-polarization modes, especially for the snow with less liquid water content.

1. INTRODUCTION

Information on spatial and temporal characteristics of the seasonal snow cover are critical to the radiation and water balances in hydrology, water management, and land surface climate [1–12, 25, 26]. Snow and alpine glaciers are important contributors to runoff in rivers and to ground-water recharges. Besides, due to the positive feedback of snow and ice surface albedo, they are also considered as key indicators of some consequences of global warming. In hydrological research, modeling and forecasting of snow-melt runoff require timely information about snow properties and their temporal and spatial variability. Assessment of water recourses in snowpack through

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conventional snow survey is often limited in scale and labor-intensive due to its remote location and inaccessibility. Therefore, snowpack is usually monitored using remote sensing techniques [9–12].

Within a snowpack, as temperature increases, water in liquid form co-exists with solid ice crystal. The volume percentage of liquid water in a snowpack is called snow liquid water content (SLWC) or simply snow wetness. Increasing SLWC is a precursor of snow melting. SLWC is a progress indicator of the snowmelt and is an essential factor for the transport of pollution from snow dumps. It is also a critical factor for estimating the mechanical strength of a snow-pack, thus affects transportation in cold regions. SLWC can affect the interaction between electromagnetic waves with snow pack and is recognized as an important factor when interpreting remotely sensed snow pack in microwave region.

However, measurement of snow wetness is usually difficult, especially on a large scale. Various methods for assessing and monitoring the liquid water content in snow cover have been proposed [13–21]. Retrieval of SLWC from both active and passive microwave remote sensing measurement has been investigated extensively [15–21]. Passive microwave remote sensing data have been used to map snow wetness and to estimate snow water equivalent [20–22]. However, due to the limited spatial resolution of microwave radiometers, their use is limited to large scale investigations. Synthetic aperture radar (SAR) enables the detection of wet snow independent of cloud cover, which can map snow wetness day and night with sufficient spatial resolution and effectively retrieve the liquid water content in snow cover [14–21].

This paper develops a backscattering model of multilayered lossy medium that can be used to investigate the backscattering properties of snowpack. The multilayered backscattering model calculates the total surface reflection coefficients of a snowpack for both horizontal and vertical co-polarizations. The total reflection coefficients are then used in the Integral Equation Method (IEM) ([20, 23–25]) to compute the backscattering coefficients. The lossy multilayered medium backscattering model has the potential to retrieval the liquid water content in snow-pack from the radar backscattering coefficients.

2. MODEL FORMULATION

2.1. Reflection from a Multilayered Lossy Medium

First, let’s consider a three-layer model with lossy media, as shown in Fig. 1, the permittivity of the three media are denoted by \( \varepsilon_1 \), \( \varepsilon_2 \), and \( \varepsilon_3 \), respectively. Layer 1 has any thickness but layer 3 is semi-infinite,
the thickness of medium 2 is \( d_2 \). Now let’s consider the reflection of an electromagnetic (EM) wave from each layer. The plane wave is incident from layer 1 and there is no EM wave reflection from the bottom of layer 3 since it is assumed to be semi-infinite. According to the geometry optics, the total reflection coefficient at the first surface (interface between layer 1 and layer 2) is given by

\[
\tilde{R}_{1,2} = R_{1,2} + T_{1,2} R_{2,3} T_{2,1} e^{-2K_{e2}d_2 / \cos \theta_2} e^{2ik_{2z}d_2} \\
+ T_{1,2} R_{2,3}^2 R_{2,1} T_{2,1} e^{-4K_{e2}d_2 / \cos \theta_2} e^{4ik_{2z}d_2} + \ldots 
\]  

(1)

where \( R_{m,n} \) is the reflection coefficient in layer \( m \) at the interface between layer \( m \) and layer \( n \), \( m, n = 1, 2, 3 \); \( T_{m,n} \) the transmission coefficient from layer \( m \) to layer \( n \); \( \theta_2 \) the refraction angle at the interface 1 and also the incidence and reflection angle at interface 2 (see Fig. 1). \( k_{2z} = k_2 \cos \theta_2, k_2 = k \sqrt{\varepsilon_2} \), where \( k \) is the wave number in free space; \( K_{e2} \) is the extinction coefficient of layer 2 [21]. The first term in the above equation is the single reflection from the first interface. The second term is the transmitted and reflected from the second interface with one internal reflection from interface 2, the third term is the term with two internal reflections from interface 2 and one reflection from interface 1, and so forth. The infinite series in (1) is the consequence of multiple internal reflections and transmissions in layer 2 of the lossy three-layer model. Thus, Equation (1) can be rewritten as

\[
\tilde{R}_{1,2} = R_{1,2} + T_{1,2} R_{2,3} T_{2,1} e^{2d_2(ik_{2z} - K_{e2} / \cos \theta_2)} \\
\left( 1 + R_{2,1} R_{2,3} e^{2d_2(ik_{2z} - K_{e2} / \cos \theta_2)} + \ldots \right) \\
= R_{1,2} + \frac{T_{1,2} R_{2,3} T_{2,1} e^{2d_2(ik_{2z} - K_{e2} / \cos \theta_2)}}{1 - R_{2,1} R_{2,3} e^{2d_2(ik_{2z} - K_{e2} / \cos \theta_2)}} 
\]  

(2)
In the previous derivation, $|R_{2,1} R_{2,3} e^{2d_2 (ik_{2z} - K_{e2}/\cos \theta_2)}| < 1$ has been used. Now, let’s consider a four-layer model, with layer 4 as semi-infinite. The second layer, the third layer, and the fourth layer form a three-layer model, just as is shown in Fig. 1 and discussed above. The surface reflection coefficient at the interface 2 is calculated similarly using the 3-layer reflection coefficient formula (Equation (2)), i.e.,

$$\tilde{R}_{2,3} = R_{2,3} + \frac{T_{2,3} R_{3,4} T_{3,2} e^{2d_3 (ik_{3z} - K_{e3}/\cos \theta_3)}}{1 - R_{3,2} R_{3,4} e^{2d_3 (ik_{3z} - K_{e3}/\cos \theta_3)}}$$  \hfill (3)$$

With the surface reflection coefficient determined, the combination of layer 3 and layer 4 is treated as an equivalent semi-infinite layer with equivalent single reflection coefficient given by Equation (3). Now the equivalent semi-infinite layer (combination of layer 3 and layer 4), layer 2, and layer 1 form a second 3-layer model. The total surface reflection coefficient is thus determined as

$$\tilde{R}_{1,2} = R_{1,2} + \frac{T_{1,2} \tilde{R}_{2,3} T_{2,1} e^{2d_2 (ik_{2z} - K_{e2}/\cos \theta_2)}}{1 - R_{2,1} \tilde{R}_{2,3} e^{2d_2 (ik_{2z} - K_{e2}/\cos \theta_2)}}$$  \hfill (4)$$

Now consider an $N$-layer model. To calculate the total surface reflection coefficient, we consider three steps:

Step 1: Every 3 layers from the bottom layer is treated as a 3-layer model, the surface reflection coefficient at the surface of the second layer from bottom is calculated using Equation (2);

Step 2: Then the bottom two layers is treated as an equivalent semi-infinite layer with equivalent surface reflection coefficient obtained in Step 1;

Step 3: Repeat Step 1 and Step 2 until the top two layers of the $N$-layer model and the last equivalent semi-infinite layer form the third layer. Finally, the total surface reflection coefficient of the $N$-layer model is calculated using Equation (2). We call this process bottom-up layer stripping.

Based on the above procedure, for an $N$-layer medium, the reflection coefficient at the interface between layer $j$ and layer $j+1$ is

$$\tilde{R}_{j,j+1} = R_{j,j+1} + \frac{T_{j,j+1} \tilde{R}_{j+1,j+2} T_{j+1,j} e^{2d_{j+1} (ik_{j+1,z} - K_{e,j+1}/\cos \theta_{j+1})}}{1 - R_{j+1,j} \tilde{R}_{j+1,j+2} e^{2d_{j+1} (ik_{j+1,z} - K_{e,j+1}/\cos \theta_{j+1})}}$$  \hfill (5)$$

Furthermore, by using the identities $T_{ij} = 1 + R_{ij}$ and $R_{ij} = -R_{ji}$, Equation (5) can be simplified as

$$\tilde{R}_{j,j+1} = \frac{R_{j,j+1} + \tilde{R}_{j+1,j+2} e^{2d_{j+1} (ik_{j+1,z} - K_{e,j+1}/\cos \theta_{j+1})}}{1 + R_{j,j+1} \tilde{R}_{j+1,j+2} e^{2d_{j+1} (ik_{j+1,z} - K_{e,j+1}/\cos \theta_{j+1})}}$$  \hfill (6)$$
2.2. A Lossy Multilayered Backscattering Model of Snow

The natural snow is a dispersive medium of a mixture of ice particle, water, and air, as shown in Fig. 2(a). Considering the difficulty and complexity in specifying the snow particles size and size distribution and shape, radiative transfer modeling of electromagnetic waves in snow is often too complicated to obtain satisfying results. To avoid this bottleneck using complicated radiative transfer modeling to obtain the volume scattering in the snow, we assume that a single layer of dispersive snow particles, liquid water content, and air is radiatively equivalent to two layers of continuum medium: the top layer is ice and liquid water content and the bottom is just air. This means that a layer of snow is separated into ice layer and air layer, but the total volume is equal to the volume of the original snow layer. Considering a 2D slab model, a snowpack of depth \(d\) above the soil surface can be modeled as a lossy multilayered medium (as shown in Fig. 2(b)). Simplifications are taken as follows:

1) The snowpack is divided evenly into \(N\) layers of snow. Each snow layer is then radiatively equivalent to a top layer of a mixture of ice and water (without air and void) with mixing dielectric constant \(\varepsilon_r\), and a bottom layer of air only. Thus, totally \(2N\) layers are formed, with odd layers being ice and liquid water; and even layers being just air.

2) Each layer of ice and liquid water content is assumed to have a uniform mixed dielectric constant. The snow grain size and size distribution can be of any value and any form.
In the above multilayered snow model, the thickness $d_n$ of each odd layer is equal to $d_o$, while that of each even layer is equal to $d_e$. According to Fig. 2(b), we have the total thickness

$$d = N(d_o + d_e)$$

(7)

The snow porosity is defined here as the volume percentage of voids in snow to the total volume of snow and is denoted by $\phi$. Since the total volume of the snow is equal to the sum of the air and the mixture of water and ice particles, we have

$$d_o = (1 - \phi)d/N$$

(8)

$$d_e = \phi d/N$$

(9)

In addition, the total reflection coefficients obtained from the multilayered snow model can be calculated iteratively using Equation (6). The variation of the total reflection coefficients will become smaller with increasing $N$. When the total reflection coefficient is saturated, i.e., the difference between $N$ and $N + 1$ layers is smaller than a preset small number, calculation stops the corresponding $N$ and thus the corresponding thickness of snow pack is called the thickness of the active scattering layer. For specific snow pack and wavelength and incidence geometry, the thickness of the active scattering layer is fixed. The number of layers from top surface of the snow pack downward that makes the calculated reflection coefficient saturated depends on the selection of $d$. The smaller the $d$ is, the more calculation is required but more accurate thickness of the active scattering layer can be found. If the thickness of the snowpack is less than the active scattering layer, the bottom homogeneous soil layer will contributes to the total reflection coefficient calculation. If the snowpack is thicker than the active scattering layer, only the top part of the snowpack contributes to the reflection coefficient.

2.3. Combination of Multilayer Model and IEM Model for Radar Backscattering Coefficient Calculation

The radar backscattering coefficient $\sigma_o^2(\theta_i)$ from a semi-infinite snowpack can be calculated by the IEM model. The IEM Model [20, 23, 24] offers a promising alternative approach for the retrieval of soil moisture, snow liquid water content, and surface roughness from active microwave data since the model is valid for a wider range of surface roughness conditions when compared to other earlier theoretical models. However, using IEM to calculate the backscattering coefficient, only the surface reflection coefficients are used and the volumetric scattering of snow is ignored. To improve the situation, we replace the surface reflection coefficients in
the IEM model, which was obtained from a single-layer model, by the total reflection coefficients obtained from the multilayered snow model, which is calculated iteratively using Equation (6). According to the multilayered snow model shown in Fig. 2 and Equation (6), the total reflection coefficient of a snowpack will include both the air-snow surface scattering and the volumetric scattering within the snow. Then, the radar backscattering coefficient calculated by the IEM model with the surface reflection coefficients replaced by the total reflection coefficients using the multilayered snow model include both surface scattering and volume scattering contributions. Therefore, for a snowpack that is above ground surface (soil layer), the total backscattering coefficient at the surface of a snowpack becomes

$$
\sigma_{t}^{o}(\theta_{i}) = \sigma_{s}^{o}(\theta_{i}) + \frac{\cos \theta_{i}}{\cos \theta_{t}} T_{as}^{2} e^{-2K_{d}/\cos \theta_{t}} \sigma_{sg}^{o}(\theta_{t})
$$

(10)

where $$\sigma_{s}^{o}(\theta_{i})$$ is the backscattering coefficients from the air-snow interface. The second term in the right hand side of Equation (1) is the volume scattering within the snow layer, which can be calculated using the above lossy multilayered snow model. $$\sigma_{sg}^{o}(\theta_{t})$$ is the backscattering coefficient from the snow-ground interface, which is attenuated by the snow layer; $$\theta_{i}$$ and $$\theta_{t}$$ are local incidence and refraction angles at the air-snow interface; $$T_{as}$$ is Fresnel power transmissivity at the air-snow interface. The $$\sigma_{sg}^{o}(\theta_{t})$$ can also be obtained using IEM model. The co-polarization backscattering coefficients from the IEM model are given by [20, 23]

$$
\sigma_{pp}^{o} = \frac{k^2}{2} \exp \left( -2k_{z}^{2} \sigma^2 \right) \sum_{n=1}^{\infty} \sigma^{2n} |I_{pp}^{n}|^2 \frac{W_{n}(-2k_{x}, 0)}{n!}
$$

(11)

where $$p = h$$ (horizontal) or $$v$$ (vertical) polarization and

$$
I_{pp}^{n} = (2k_{z})^{n} f_{pp} \exp \left( -k_{z}^{2} \sigma^2 \right) + k_{z}^{n} F_{p}(k_{x})/2,
$$

(12a)

$$
f_{vv} = 2R_{||}/\cos \theta
$$

(12b)

$$
f_{hh} = -2R_{\perp}/\cos \theta
$$

(12c)

$$
\Phi_{v} = F_{vv}(-k_{x}, 0) + F_{vv}(k_{x}, 0)
$$

(12d)

$$
\Phi_{h} = F_{hh}(-k_{x}, 0) + F_{hh}(k_{x}, 0)
$$

(12e)

where $$\sigma$$ is the surface Root Mean Square (RMS) height, $$k$$ the wave number of radar in air, $$k_{z} = k \cos \theta$$, $$k_{x} = k \sin \theta$$, and $$\theta$$ the incidence
angle. $R_{∥}$ and $R_{⊥}$ are the reflection coefficients of vertical and horizontal polarization, respectively. It is there two terms in the IEM model that link the backscattering coefficient with reflection coefficients that is our focus in the present study. $W^n(k_x, k_y)$ is the Fourier transform of the $n$th power of a known surface correlation function which is calculated by

$$W^n(k_x, k_y) = \frac{1}{2\pi} \int \int \rho^n(x, y) \exp(jk_xx + jk_yy)dxdy$$

where $\rho(x, y)$ is the surface correlation function. The surface correlation function, such as Gaussian and exponential surface correlation functions, includes the random surface height and correlation length, which can be used to describe the soil surface conditions.

3. ANALYSIS AND VALIDATION

A dry snowpack is taken as the reference, and its parameters are listed in Table 1. Backscattering coefficients versus frequency are shown in Fig. 3 for like polarization. The results are calculated using the lossy multilayered backscattering model presented above at 5, 10, and 15 GHz, and shows the incidence angle dependence of backscattering coefficients. The snow layer permittivity relative to vacuum is around 1.5 so that scattering from the air-snow interface is very small. Although the snow layer is 1 m thick, loss through

<table>
<thead>
<tr>
<th>Table 1. Snow and soil parameters for backscattering model.</th>
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<tbody>
<tr>
<td><strong>Parameters</strong></td>
</tr>
<tr>
<td>----------------</td>
</tr>
<tr>
<td>Frequency</td>
</tr>
<tr>
<td>Snow pack properties</td>
</tr>
<tr>
<td>Snow depth</td>
</tr>
<tr>
<td>Porosity</td>
</tr>
<tr>
<td>Snow permittivity</td>
</tr>
<tr>
<td>RMS height</td>
</tr>
<tr>
<td>Correlation length</td>
</tr>
<tr>
<td>Soil properties</td>
</tr>
<tr>
<td>Soil permittivity</td>
</tr>
<tr>
<td>RMS height</td>
</tr>
<tr>
<td>Correlation length</td>
</tr>
</tbody>
</table>
The backscatter calculations for wet snow were based on the average measured liquid water content of 4% and an average grain radius of 0.75 mm. For the case of wet snow, the contribution of backscattering from the snow-ground interface can be neglected, and backscattering at the air-snow interface dominates. From Fig. 4, it can be seen that the difference between the simulations and the measurements for dry snow are less than 1 dB for the range of incidence angle $\theta$ ($10^\circ$–$70^\circ$) and they agree well. For wet snow, the difference between the simulated results and the measured values are found to become large when the incidence angle is large, especially at incidence angles above $30^\circ$. In addition, for both dry snow and wet snow, the measured values are more agreement
Table 2. Snow and soil parameters for VV polarization.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Units</th>
<th>Dry snow case</th>
<th>Wet snow case</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>GHz</td>
<td>5.3</td>
<td>5.3</td>
</tr>
<tr>
<td>Snow pack properties</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Snow depth</td>
<td>mm</td>
<td>2000</td>
<td>500</td>
</tr>
<tr>
<td>Porosity</td>
<td>%</td>
<td>66.7</td>
<td>50</td>
</tr>
<tr>
<td>Liquid water content</td>
<td>%</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>Layer permittivity</td>
<td></td>
<td>1.53 – j0.0002</td>
<td>2.61 – j0.38</td>
</tr>
<tr>
<td>RMS height</td>
<td>mm</td>
<td>4.4</td>
<td>4.4</td>
</tr>
<tr>
<td>Correlation length</td>
<td>mm</td>
<td>116</td>
<td>116</td>
</tr>
<tr>
<td>Soil surface properties</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ground permittivity</td>
<td></td>
<td>16.3 – j3.7</td>
<td>16.3 – j3.7</td>
</tr>
<tr>
<td>RMS height</td>
<td>mm</td>
<td>15.8</td>
<td>15.8</td>
</tr>
<tr>
<td>Correlation length</td>
<td>mm</td>
<td>117</td>
<td>117</td>
</tr>
</tbody>
</table>

with the simulated results than with the modeled results in Nagler and Rott [15].

4. CONCLUSION

A lossy multilayered medium backscattering model has been presented in this paper. Reflection in a lossy multilayered medium has been investigated, and the generalized reflection coefficients for horizontal and vertical co-polarization are obtained. Moreover, the lossy multilayered backscattering model of snow has been constructed and the numerical analysis of backscattering coefficients has been performed. The results from the comparison indicate that the difference between the simulations and the measurements for dry snow are less than 1 dB for the range of incidence angle $\theta$ (10°–70°) and the difference between the simulated results and the measured values for wet snow are found to become large when the incidence angle is large, especially at incidence angles above 30°.

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