Assessment of the accuracy of snow surface direct beam spectral albedo under a variety of overcast skies derived by a reciprocal approach through radiative transfer simulation

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With radiative transfer simulations it is suggested that stable estimates of the highly anisotropic direct beam spectral albedo of snow surface can be derived reciprocally under a variety of overcast skies. An accuracy of ±0.008 is achieved over a solar zenith angle range of θ₀ = 74° for visible wavelengths and up to θ₀ = 63° at the near-infrared wavelength λ = 862 nm. This new method helps expand the database of snow surface albedo for the polar regions where direct measurement of clear-sky surface albedo is limited to large θ₀’s only. The enhancement will assist in the validation of snow surface albedo models and improve the representation of polar surface albedo in global circulation models. © 2003 Optical Society of America

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1. Introduction

Albedo is the ratio of the reflected shortwave irradiance at the surface to the incident shortwave irradiance. Because surface albedo is an important geophysical parameter that determines shortwave energy exchange at the Earth’s surface, accurate albedo estimates are required to determine the surface energy balance in global circulation models (GCMs). Snow surface albedo is particularly important because snow cover—having a high reflectance, occupying ~46% of the land in the Northern Hemisphere winter as well as vast areas of winter sea ice, and exerting a strong positive feedback effect on the surface energy budget—exhibits a notorious variation of surface albedo with the aging of the snow pack, variation of solar zenith angle (θ₀), and changes in the spectral composition and angular pattern of the incoming radiation under various cloud cover conditions.

Under clear skies, the snow surface albedo increases with θ₀ because of the strong forward scattering from the snow grains in the snow cover. Cloud cover can change the effective incidence angle. Clouds also absorb more strongly in the infrared than in the visible: Therefore under cloudy conditions a greater portion of the incident irradiance is at visible wavelengths where the snow and ice albedo is larger. Consequently, the snow and ice surface all-wave albedo under cloudy skies is typically 4–11% larger than clear-sky values when the solar incidence angle is not extremely large.

To reduce the uncertainty of snow surface albedo in GCM models and snow and ice physical and biological process models, it is more appropriate to evaluate snow surface all-wave albedo through integration of direct beam spectral albedo over the full range of incidence angles with the pattern of directional downwelling spectral radiance as a weighting function. The advantage of such an approach is that the direct beam spectral albedo is intrinsic only to the surface condition.

With radiative transfer (RT) models we are able to produce a surface direct beam spectral albedo over a full range of incidence angles for certain snow-covered surface types. However, a thorough validation of such models is hampered because of the
difficulty in measuring the surface direct beam spectral albedo for a wide range of incident angles. This difficulty is caused by two facts. One is the lack of clear-sky conditions for most areas. For example, we had only five clear-sky sea ice stations during our 45-day cruise in the Southern Ocean in February and March 2000. The other is the narrow range of solar incidence angles that occur during measurement. This is especially true when the measurements are made at high latitudes and within a few hours during a day. In the field of snow and sea ice investigation, there are few direct beam spectral albedo data available that cover a wide range of snow and ice types and a full range of incident angles.

To expand the snow surface albedo database for the sake of improving snow surface albedo representation in GCMs and other physical and biological process models, we experimented with a new method to derive a surface direct beam spectral albedo for a wide range of solar incidence angles using surface directional spectral reflectance measurements made under overcast conditions. The derivation is based on the reciprocity between the direct beam spectral albedo and the hemispherical-directional spectral reflectance values. In this study we first provide a theoretical error analysis of the reciprocity between the hemispherical-directional spectral reflectance measured under overcast conditions and the direct beam spectral albedo for clear skies. We then use a multilayer zenith- and azimuth-dependent RT model to simulate both snow surface direct beam spectral albedo under clear skies and directional spectral reflectance under various overcast conditions. Finally, we assess the feasibility of the reciprocal method in practical use by comparing the two sets of simulation results.

2. Theoretical Error Analysis of the Reciprocal Method

In optical reflectometry, there is reciprocity between the hemispherical-directional spectral reflectivity $R_{\lambda}(\theta, \phi)$ and the clear-sky direct beam spectral albedo $\alpha_{b,\lambda}(\theta_0, \phi_0)$ (Fig. 1):

$$R_{\lambda}(\theta, \phi) = \alpha_{b,\lambda}(\theta_0, \phi_0), \quad \text{for } \theta = \theta_0, \phi = \phi_0$$

where the incident radiance from the hemispherical illumination is uniform in all directions. Here the subscript $\lambda$ represents wavelength dependence and $b$ denotes beam incidence. Hereafter $\theta$ and $\phi$ represent zenith and azimuth angles with the subscript $x$ replaceable by $v$ for viewing direction, 0 for beam incidence, and $i$ for downwelling diffuse incidence in general. However, these distinctions blur when the bidirectional reflectance quantities originally defined under clear skies are adopted for overcast conditions and some basic reciprocal relations are pursued. To avoid confusion of the subscripts of the directional variables, we follow the convention for the directional quantities: The directional variables in parentheses are related to incidence if they appear before the semicolon, and those after it are related to reflection.

Note that there is an azimuthal dependency of the two quantities in Eq. (1). This implies that Eq. (1) holds even for surfaces that exhibit a variation of direct beam spectral albedo with the azimuth of the beam incidence if the isotropic condition of the incident radiance is met. For snow surface, such a variation can be caused by the formation of snow dunes, barchans, and sastrugi at the snow surface.

Although the ground-level sky diffuse light under overcast skies seems to be isotropic on the basis of visual observation, it may actually be anisotropic. Therefore whether we can produce accurate estimates of snow surface direct beam spectral albedo using the reciprocal method for typical overcast conditions is a question we must answer to assess the feasibility of the reciprocal method in practical use. To answer this question, we first conduct a theoretical analysis of errors in the derived direct beam spectral albedo when the sky diffuse light is arbitrarily anisotropic.

According to Siegel and Howell, the hemispherical-directional spectral reflectance under overcast conditions can be expressed as

$$R_{\lambda}(\theta_0, \phi_0) = \frac{\pi \iint L^i_{\lambda}(\theta, \phi) f_{r,\lambda}(\theta, \phi; \theta_0, \phi_0) \cos \theta \sin \theta_0 d\theta_0 d\phi_0}{E^i_{\lambda}},$$

where $f_{r,\lambda}$ is the bidirectional reflectance distribution function (BRDF). The downwelling diffuse irradiance $E^i_{\lambda}$ equals the hemispherical integral of downwelling diffuse radiance $L^i_{\lambda}$:

$$E^i_{\lambda} = \iint L^i_{\lambda}(\theta, \phi) \cos \theta \sin \theta_0 d\theta_0 d\phi_0.$$
We can evaluate $R_\lambda(\theta_c)$ by averaging $R_\lambda(\theta_v, \phi_v)$ over the $\phi_v$ terms:

$$R_\lambda(\theta_v) = \frac{\int_0^{2\pi} R_\lambda(\theta_v, \phi_v) d\phi_v}{2\pi}$$

This expression is especially useful for measurement because the averaging can significantly reduce the impact of random errors. Because direct beam spectral albedo also does not depend on azimuth angle for most homogeneous natural surfaces, we use $R_\lambda(\theta_v)$ to compare with $\alpha_{b,\lambda}(\theta_0 = \theta_v)$ throughout the rest of the paper.

Similarly, BRDF depends on the relative azimuth angle between incidence and viewing directions instead of individual azimuth angles for most homogeneous natural surfaces. Hereafter, we primarily use bidirectional reflectance function (BRF) instead of BRDF to describe the anisotropic nature of surface reflectance. BRF is a dimensionless quantity resulting from when the BRDF is multiplied by a scaling factor $\pi$. The advantage of our using BRF is that BRF values generally have a magnitude similar to the surface spectral albedo. We can introduce the azimuthally averaged BRF $L_\lambda^i(\theta_i, \phi_i; \theta_v, \phi_v)$ instead of $f_{\lambda,\lambda}(\theta_v, \phi_v)$ throughout the expression of Eq. (6) as follows:

$$L_\lambda^i(\theta_i, \phi_i; \theta_v, \phi_v) = \int_0^{2\pi} f_{\lambda,\lambda}(\theta_v, \phi_v; \theta_i, \phi_i) d\phi_v$$

Also, we introduce azimuthally averaged sky diffuse spectral radiance,

$$L_\lambda^i(\theta_i) = \frac{\int_0^{2\pi} L_\lambda^i(\theta_i, \phi_i) d\phi_i}{2\pi}$$

and express the direct beam spectral albedo $\alpha_{b,\lambda}$ in terms of the BRDF:

$$\alpha_{b,\lambda}(\theta_0 = \theta_v) = \int_0^{2\pi} f_{\lambda,\lambda}(\theta_v, \phi_v; \theta_i, \phi_i) \times \cos \theta_i \sin \theta_i d\phi_i d\phi_v.$$  (7)

Using Eqs. (4)–(7) we obtain the discrepancy between azimuthally averaged hemispherical-directional reflectance and the direct beam albedo:

$$\Delta \alpha_{b,\lambda}(\theta_0 = \theta_v) = R_\lambda(\theta_v) - \alpha_{b,\lambda}(\theta_0 = \theta_v)$$

Introducing $L_\lambda^i = \frac{E_{\lambda}}{E_{\lambda,\lambda}} \pi$, $\Delta L_\lambda^i(\theta_i) = L_\lambda^i(\theta_i) - L_{\lambda,\lambda}^i$, and $R_\lambda(\theta_v; \theta_i) = R_\lambda(\theta_v; \theta_i) - \alpha_{b,\lambda}(\theta_v)$, we can further express Eq. (8) as follows:

$$\Delta \alpha_{b,\lambda}(\theta_0 = \theta_v) = \frac{2\pi}{E_{\lambda,\lambda}} \int_0^{\pi/2} \left[ \frac{L_\lambda^i + \Delta L_\lambda^i(\theta_i)}{R_\lambda(\theta_v; \theta_i) \cos \theta_i \sin \theta_i} \right] \times \frac{R_\lambda(\theta_v; \theta_i) \cos \theta_i \sin \theta_i d\theta_i}{2\pi}$$

$$- 2 \int_0^{\pi/2} \frac{R_\lambda(\theta_v; \theta_i) \cos \theta_i \sin \theta_i d\theta_i}{2\pi}$$

$$= \frac{2\pi}{E_{\lambda,\lambda}} \int_0^{\pi/2} \frac{\Delta L_\lambda^i(\theta_i) R_\lambda(\theta_v; \theta_i) \cos \theta_i \sin \theta_i d\theta_i}{2\pi}$$

$$+ \frac{2\pi \alpha_{b,\lambda}(\theta_v)}{E_{\lambda,\lambda}} \int_0^{\pi/2} \Delta L_\lambda^i(\theta_i) \cos \theta_i \sin \theta_i d\theta_i.$$  (9)

Based on the definition of $\Delta L_\lambda^i(\theta_i)$ and Eqs. (3) and (6),

$$\int_0^{\pi/2} \Delta L_\lambda^i(\theta_i) \cos \theta_i \sin \theta_i d\theta_i$$

$$= \int_0^{\pi/2} L_\lambda^i(\theta_i) \cos \theta_i \sin \theta_i d\theta_i$$

$$- \int_0^{\pi/2} \frac{E_{\lambda,\lambda}}{2\pi} \frac{E_{\lambda,\lambda}}{2\pi} = 0.$$  (10)
Thus we finally obtain the following relation:
\[ \Delta \alpha_{b,\lambda}(0) = \theta_0 \]
\[ = \frac{2\pi}{E_{\text{diff}\lambda}} \int_{0}^{\pi} \Delta L^i_{\lambda}(\theta) \Delta R_{\lambda}(\theta, \theta_0) \cos \theta \sin \theta \, d\theta. \]

(11)

This means that the error \( \Delta \alpha_{b,\lambda}(0) = \theta_0 \) depends on the anisotropy of both sky diffuse light and surface BRF. When sky diffuse radiation is uniform, all \( \Delta L^i_{\lambda}(\theta) \) values are zero, and \( \Delta \alpha_{b,\lambda}(0) = \theta_0 \) must be zero. This is exactly the condition for the reciprocity in Eq. (1) to hold. When \( R_{\lambda}(\theta, \theta_0) \) is constant, the surface is Lambertian. \( \Delta \alpha_{b,\lambda}(0) = \theta_0 \) must be zero too. From the viewpoint of statistics, \( \Delta \alpha_{b,\lambda}(0) = \theta_0 \) is obviously proportional to the covariance of the zenith-dependent and azimuthally averaged sky diffuse radiance and surface BRF. Consequently, the error would be small when the sky diffuse light and surface BRF vary randomly. For general overcast conditions, therefore, we focus on the relations among \( \Delta L^i_{\lambda}(0) = \theta_0 \), \( R_{\lambda}(\theta, \theta_0) \), and \( L^i_{\lambda}(\theta) \) only when sky diffuse light and surface BRF exhibit nonrandom variations.

3. Simulation of Snow Surface Clear-Sky Direct Beam Spectral Albedo and Diffuse Sky Directional Spectral Reflectance

To study nonrandom variation of diffuse sky light and snow surface BRF patterns, we use a plane-parallel layered model to represent stratus clouds and snow cover. We use a RT model to simulate both direct beam spectral albedo under clear skies and surface hemispherical-directional spectral reflectance under various overcast conditions.

A. Models Used in Simulation

An azimuth- and zenith-dependent plane-parallel RT code \(^{26}\) is used in simulation. The code is based on doubling and adding methods \(^{27-29}\) to handle the coupling of multiple layers, each of which can be optically thick. The code takes the single-scattering albedo, asymmetry factor, and optical thickness values of individual layers as input. Given the solar incidence angle above the top layer and the albedo of an underlying surface beneath the lowest layer, the model produces a full range of angular patterns of the upwelling and downwelling spectral radiance at the top and bottom of the individual layers. The handling of zenith and azimuth variation of the radiation field is realized through Gaussian quadrature-based division in the zenith domain and evenly distributed second divisions in the azimuth domain. Fourier decomposition of the RT equation in the azimuth is performed by fast cosine expansion \(^{30}\) of the single-scattering phase function in the azimuth domain to save computational time. Specifically, we subdivide each of the up and down hemispheres into 16 (zenith) by 64 (azimuth) divisions to study the patterns of snow surface BRF under clear skies and downwelling diffuse radiance and hemispherical-directional reflectance at the snow surface under overcast skies. We also use this model to calculate the snow surface direct beam spectral albedo through hemispherical integration of BRF.

1. Snow Model

To investigate the relation between the error in the direct beam spectral albedo estimation and the anisotropy of surface reflectance, the same model is used to simulate the snow surface BRF and azimuthally averaged BRF patterns. The specifications of the model are listed in Table 1. The selection of snow grain radii \( r_s \) is conventional because 0.2 and 1.0 mm are traditionally considered as the fine-grained old snow and old snow near the melting point. Hereafter we call them medium and large snow grain sizes for simplicity. A snow optical thickness \( \tau_s \) of 100 is also adequate because snow pack may be semi-infinite although it is often optically thick.

The selection of a snow particle model is a challenge. Recently, Mishchenko et al. \(^{36}\) compared three basic shape models, i.e., irregular, spherical, and hexagonal, for snow particles to study the impact of nonsphericity of snow particles on their scattering properties. \(^{31}\) The resulting differences in the asymmetry factor are large. Field observations of snow pack suggest that snow particles tend to become rounded because of metamorphism as they age. \(^{32,33}\) Under near-melting conditions, they are further sintered together to form composite grains because of melt and refreeze conditions. \(^{34}\) These facts imply that a spherical model, irregular model, or something in between is appropriate for medium and large snow grains.

To simplify the simulation, we use a spherical model in this investigation for several reasons. First, simulation of single-scattering properties of spherical particles is well established, and the computer codes are available to us. \(^{35,36}\) Second, our focus in this study is the snow surface direct beam spectral albedo. The spherical model that has been proven appropriate for simulation of snow surface albedo \(^{14,37}\) is thus used in this study. Third, our particular interest in this research is the accuracy of the reciprocally derived albedo. As indicated by Kokhanovsky and Macke, \(^{38}\) a spherical-shape model generates the largest asymmetry factor among all convex large nonspherical parti-
Table 1. Specifications of Snow and Cloud–Snow Coupled Models Used in the Simulation

<table>
<thead>
<tr>
<th>Snow Model</th>
<th>One-layer RT Model</th>
<th>Cloud–Snow Coupled Model</th>
<th>Two-layer RT Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Snow grain radius ($r_s$)</td>
<td>0.2, 1.0 mm</td>
<td>Equivalent radius of cloud droplets ($r_c$)</td>
<td>4.0, 8.0, 14.0 μm</td>
</tr>
<tr>
<td>Snow optical thickness ($\tau_s$)</td>
<td>100</td>
<td>Cloud droplet size distribution parameter ($\gamma$)</td>
<td>8</td>
</tr>
<tr>
<td>Wavelengths ($\lambda$)</td>
<td>415, 500, 610, 665, 862, 2250 nm</td>
<td>Cloud optical thickness ($\tau_c$)</td>
<td>5, 10, 15, 20, 25, 30, 40, 60</td>
</tr>
<tr>
<td>Solar incidence angles (for BRF comparison with results from reciprocity) ($\theta_o$)</td>
<td>5.9°, 13.5°, 21.1°, 28.6°, 36.0°, 43.2°, 50.1°, 56.8°, 63.1°, 68.9°, 74.3°, 79.0°, 83.0°, 86.1°, 88.4°, 89.7°</td>
<td>Solar incidence angles above cloud ($\theta_o$)</td>
<td>30°, 65°, 80°</td>
</tr>
<tr>
<td>Solar incidence angles (for BRF investigation) ($\theta_i$)</td>
<td>0°, 30°, 65°, 80°</td>
<td>Bottom snow layer</td>
<td>See snow model to the left except for solar incidence angles</td>
</tr>
<tr>
<td>Bottom albedo (Lambertian)</td>
<td>0.2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Diffuse Sky Model

We use the same RT model for a two-layered stratus cloud and snow-coupled model [Fig. 2(b)]. The details of the two-layer model are also listed in Table 1.

To study the impact of various sky conditions on the accuracy of the derived direct beam spectral albedo from the reciprocal method, 72 sky conditions are formed from combinations of eight cloud optical thicknesses ($\tau_c$), three equivalent radii of cloud droplets ($r_c$), and three solar incidence angles above clouds ($\theta_o$). The selection of $\tau_c$ is based on a ground-based remote sensing study of clouds in the Arctic, as well as a 25-month database of stratus cloud properties at a Southern Great Plains site. The mean $\tau_c$ in both investigations is around 25–30 with a standard deviation of 13–20. Therefore $\tau_c$ values that range from 5 to 60 are used in the simulation. The selection of $r_c$’s ($r_c = 4, 8, 14 \mu m$), the size distribution parameter ($\gamma = 8$), and the wavelengths are identical to those in Leontieva and Stamnes so that the single-scattering albedo and asymmetry factor of the cloud layer at the given wavelengths can be directly obtained from their results. Three above-cloud solar incidence angles ($\theta_o = 30°, 65°$, and $80°$) are applied to the model to study the $\theta_o$ effect on the accuracy of the estimates.

The bottom snow layer is identical to that used in the simulation of the clear-sky snow surface albedo and the BRF.

The output of the simulation includes the hemispherical-directional reflectance pattern and the downwelling sky diffuse spectral radiance pattern at the cloud–snow interface. The former is compared with the direct beam spectral albedo from the clear-sky snow model to assess the accuracy of the estimates from the reciprocal method. The latter is used to analyze the relation between the error in the direct beam albedo estimation and the departure of sky diffuse light from isotropy.

B. Simulation Results

1. Clear-Sky Snow Surface Albedo

For a homogeneous snow layer with an optical thickness ($\tau_s = 100$) underlain with a moderately dark
Lambertian surface (albedo of 0.2), the snow surface direct beam spectral albedo ($\alpha_{b,\lambda}$) at the six selected wavelengths were calculated for the solar zenith angles ($\theta_0$) listed in Table 1. Figure 3 shows the patterns of the direct beam spectral albedo as a function of $\theta_0$ with snow grain size ($r_s$) as a parameter. For all cases, snow surface $\alpha_{b,\lambda}$ increases with $\theta_0$. The rate of increase of $\alpha_{b,\lambda}$ increases with $\theta_0$. This non-linear feature becomes more and more conspicuous as the wavelength increases. The $\alpha_{b,\lambda}$ also increases with decreasing $r_s$ with a greater sensitivity at near-infrared (NIR) and shortwave-infrared (SWIR) wavelengths ($\lambda = 862$ and 2250 nm). Figure 4 shows the same simulation with the wavelength used as a parameter. For both medium- and large-sized snow grains ($r_s = 0.2$ and 1.0 mm), $\alpha_{b,\lambda}$ values are all high at visible wavelengths ($\lambda = 415$–665 nm), and differences among them are barely discernable. As the wavelength further increases, the spectral albedo at the NIR and SWIR wavelength regions have much lower values. Because we do not focus on variation of snow surface albedo with wavelength and snow grain size in this paper, we do not discuss the relation among them in detail. Nevertheless, we point out that the selected cases cover a wide range of snow surface direct beam spectral albedo and are representative when used to assess the accuracy of the snow surface spectral albedo derived from the reciprocal method.

2. Snow Surface Bidirectional Reflectance Function Pattern

We also derive snow surface BRF patterns for the selected cases for an intuitive analysis of the errors of the derived albedo in relation to the anisotropy of surface reflectance, although the theoretical background was discussed in Section 2.

Figure 5 shows representative BRF patterns for various cases. Figures 5(a) and 5(b) provide BRF patterns with vertical solar incidence for $r_s = 0.2$ and 1.0 mm, respectively. For both snow grain sizes, the BRF patterns are symmetric with the vertical axis and exhibit a limb-darkening pattern at the visible (500-nm) and NIR (862-nm) wavelengths and a trend of general limb brightening at SWIR (2250 nm). At visible and NIR wavelengths, the strongest differential scattering intensity does not occur at the near-surface sublayer because of multiple scattering among nonabsorbing snow grains. Consequently, viewing along a vertical path will see a brighter scattering layer deep in the snow pack than along a slant path with larger viewing zenith angles ($\theta_i$). This leads to limb darkening. At the SWIR wavelength region, stronger absorption of radiation by snow grains causes a rapid decrease of scattering intensity with snow depth. This effect favors a generally enhanced BRF with a large $\theta_i$ because more bright scatterers near the surface are seen along such a path. Figures 5(c) and 5(d) present BRF patterns...
along the principal plane at $\lambda = 500$ nm for medium and large $r_s$ terms, respectively. The principal plane is the plane defined by the vertical axis and the vector of the solar beam. Figures 5(c) and 5(d) reveal limb darkening for vertical and small $\theta_0$'s ($0^\circ$–$30^\circ$) and strong forward scattering for large $\theta_0$'s ($65^\circ$–$80^\circ$). These patterns are similar to those presented in Li et al., except that the snow optical thickness is not semi-infinite in our study. Figures 5(e) and 5(f) provide full BRF patterns at $\lambda = 500$ nm with a snow grain size of 1.0 mm for small $\theta_0$ ($30^\circ$) and large $\theta_0$ ($65^\circ$), respectively. The $x$ axis represents different viewing azimuth angles ($\phi_v$), and different curves represent different viewing zenith angles ($\theta_v$). For small incidence angles, limb darkening is revealed by the generally lower position of curves for larger $\theta_v$ terms than those for smaller $\theta_v$ terms. However, slant incidence causes a skewed BRF pattern that exhibits less limb darkening at the forward direction ($\phi_v \approx 0^\circ$). For large solar incidence angles, a strong scattering peak appears at large $\theta_v$ terms at the forward direction ($\phi_v \approx 0^\circ$).

For an analysis of the accuracy of the reciprocallly derived direct beam spectral albedo, we further look into the anisotropy of the azimuthally averaged snow surface spectral BRF [$R_s(\theta_0; \theta_v)$] based on Eq. (11). Figure 6 shows representative snow surface $R_s(\theta_0; \theta_v)$ patterns in relation to $\theta_v$, $\lambda$, $r_s$. Note that $R_s(\theta_0; \theta_v)$ is on a logarithmic scale. Figures 6(a) and 6(b) provide $R_s(\theta_0; \theta_v)$ patterns for small ($30^\circ$) and extra large ($80^\circ$) incidence angles, respectively. Figures 6(c) and 6(d) give a three-dimensional view of the $R_s(\theta_0; \theta_v)$ patterns at visible ($\lambda = 500$-nm) and SWIR ($\lambda = 2250$-nm) wavelengths, respectively. Their symmetric patterns also reveal the reciprocity between $R_s(\theta_0; \theta_v)$ and $R_s(\theta_0; \theta_v)$ [Eq. (5)] when the incidence and viewing zenith angles are interchanged. For small $\theta_0$ [Figs. 6(a) and 6(c)], the limb darkening at visible and NIR wavelengths and the general limb brightening at the SWIR wavelength are similar to those for vertical incidence [see Fig. 5(a)]. When $\theta_0$ is large, the $R_s(\theta_0; \theta_v)$ patterns exhibit a generally increasing trend with increasing $\theta_v$ [Figs. 6(c) and 6(d)] because of the forward-scattering nature of snow grains. When $\theta_0$ becomes extra large, the $R_s(\theta_0; \theta_v)$ patterns are extremely anisotropic [Figs. 6(b)–6(d)]. It is obvious that the extra large $\theta_0$ ($80^\circ$), the $R_s(\theta_0; \theta_v)$ patterns at $\lambda = 2250$ nm are much more anisotropic than those at shorter wavelengths. This contrast can also be seen by a comparison of Figs. 6(c) and 6(d).

On the basis of these simulation results and Eq. (11), we expect that the errors of direct beam spectral albedo derived by the reciprocal approach would be larger for extra large $\theta_v$ terms than those for small and intermediate $\theta_v$ terms if the sky diffuse light exhibits a monotonously increasing or decreasing pattern. Also, the errors for extra $\theta_v$ terms would be even larger at $\lambda = 2250$ nm because the stronger absorption of snow particles and the consequent less multiple scattering in the snow pack at this wavelength lead to stronger anisotropy in the surface bidirectional reflectance pattern.

3. Patterns of Sky Diffuse Light

Using the coupled cloud and snow model described in Subsection 3.A.2 and the specifications listed in Table 1, we simulated the patterns of sky diffuse radiance for 72 sky conditions and at the six wavelengths defined in Subsection 3.A.2 over snow packs with different snow grain sizes ($r_s = 0.2$ and 1.0 mm). To facilitate a comparison among different cases, the radiance values in this subsection are normalized with respect to the case-specific averaged spectral radiance value at the cloud–snow interface.

Figure 7 shows examples of normalized $L_d^\lambda(\theta_i; \phi_v)$ at $\lambda = 862$ nm for various $\tau_v$'s and $\theta_0$'s with $r_s = 8 \mu$m and $r_s = 1.0$ mm. The wavelength $\lambda = 862$ nm is chosen because the absorption of cloud droplets at the
wavelength is intermediate and the general relation of \( L^i_c(\theta_i, \phi_i) \) patterns with \( \tau_c \)'s and \( \theta_{oc} \)'s is representative for other cases. Figures 7(a) and 7(b) are the result of the same thin clouds (\( \tau_c = 5 \)) but with different \( \theta_{oc} \)'s (30° and 80°). The \( L^i_c(\theta_i, \phi_i) \) patterns for both cases vary not only with diffuse incoming zenith angles (\( \theta_i \)) shown as different curves (with symbols \( Z_i = x \times x \)) but also with azimuth angles (\( \phi_i \)). The differences between the two cases are (1) the \( L^i_c(\theta_i, \phi_i) \) pattern for large \( \theta_{oc} \) is less anisotropic (the ratio of maximum over minimum is 2.0) than that for small \( \theta_{oc} \) (the ratio of maximum over minimum is 3.5), and (2) there is strong peak of \( L^i_c(\theta_i, \phi_i) \) at \( \theta_i \approx 30^\circ \) that is close to the direct beam direction (\( \theta_{ci} = 30^\circ \)) whereas there is no such coincidence for \( \theta_{ci} = 80^\circ \). These differences are due to the difference in scattering path lengths when the direct beam passes through the cloud layer with different incidence angles. The larger solar incidence angle results in a longer scattering path, more attenuation, and less anisotropic diffuse light. As cloud optical thickness increases [Figs. 7(c) and 7(d)], the dependence of \( L^i_c(\theta_i, \phi_i) \) on \( \phi_i \) disappears. However, the sky diffuse light still exhibits a limb-darkening pattern, which is shown in Figs. 7(c) and 7(d) where curves of \( L^i_c(\theta_i, \phi_i) \) with larger \( \theta_i \)'s have smaller radiance values than those with smaller \( \theta_i \)'s. These examples indicate that, when the clouds are sufficiently thick, azimuthal dependence of sky diffuse light disappears.

For the purpose of an accurate assessment of the direct beam spectral albedo derived from the reciprocal approach, we focus on the anisotropy of the azimuthally averaged sky diffuse radiance \( L^i_c(\theta_i) \) as we did for snow surface BRF.

Figure 8 provides the normalized \( L^i_c(\theta_i) \) patterns as a function of \( \theta_i \) with \( \tau_c \) and \( \theta_{oc} \) as parameters, and wavelength (\( \lambda \)) varies among the graphs and is specified in the figure caption. Only patterns for \( \lambda = 500, 862, \) and 2250 nm and \( r_c = 1 \) mm are given as examples because patterns of other combinations are either similar to one of them or something between them. In the legends, t05 and t10 denote \( \tau_c = 5 \) and \( \tau_c = 10 \); and inc30, inc65, and inc80 represent \( \theta_{oc} = 30^\circ, 65^\circ, \) and \( 80^\circ \), respectively. For moderate to large cloud optical thicknesses (\( \tau_c = 15, 20, 25, 30, 40, \) and 60), the differences among the normalized \( L^i_c(\theta_i) \) patterns are extremely small. Therefore only the range and mean values are presented as the maximum, minimum, and mean.

Figures 8(a), 8(b), and 8(c) illustrate \( L^i_c(\theta_i) \) patterns for a wide cloud droplet size range (\( r_c = 4, 8, \) and 14 \( \mu \)m) with \( \lambda = 500, 862, \) and 2250 nm, respectively. For each of these graphs all 72 sky conditions defined in Subsection 3.A.2 are involved, including combinations from 6 \( \tau_c \)'s, 3 \( r_c \)'s, and 3 \( \theta_{oc} \)'s. Comparing Fig. 8(a) with Fig. 8(b), it is obvious that a variation of cloud droplet size causes only slight variations of \( L^i_c(\theta_i) \) patterns for thin clouds (\( \tau_c = 5 \)) and exerts almost no impact on the \( L^i_c(\theta_i) \) patterns for thicker clouds (\( \tau_c \geqslant 10 \)) at visible and NIR wavelengths (\( \lambda = 500 \) and 862 nm). For thin clouds (\( \tau_c = 5 \)), \( L^i_c(\theta_i) \) patterns (blue curves) vary with \( \theta_{oc} \) signif-

Fig. 8. Patterns of normalized \( L^i_c(\theta_i) \) as a function of \( \theta_i \) at (a) \( \lambda = 500 \), (b) \( \lambda = 862 \), (c) \( \lambda = 2250 \), (d) \( \lambda = 2250 \) nm for various sky conditions (\( \tau_c = 5–60, r_c = 4–14 \mu \)m, and \( \theta_{oc} = 30°–80° \)) over snow pack with \( r_s = 1.0 \) mm. The continuous curves represent mean values. The different colors represent different cloud optical thicknesses with blue for thin (\( \tau_c = 5 \)), green for moderately thin (\( \tau_c = 10 \)), and red for others (\( \tau_c = 15–60 \)). Different shades of blue and green curves denote different \( \theta_{oc} \)'s in the thin and moderately thin cloud cases. The exception is (d) where only a \( \tau_c \) range from 15 to 60 is involved and different colors represent different \( r_c \) terms. inc, incidence; cr, cloud droplet radius.
icantly. They do not exhibit a pattern of monotonous increase or decrease with $\theta_s$. For small $\theta_{oc}$ ($30^\circ$), the maximum of $L_\lambda^L(\theta)$ appears where $\theta_s \sim \theta_{oc}$. For large $\theta_{oc}$ ($65^\circ$ and $80^\circ$), the $L_\lambda^L(\theta)$ maxima are located at the middle range of $\theta_s$'s. As $\tau_c$ increases, the variation of the $L_\lambda^L(\theta)$ pattern with $\theta_{oc}$ decreases. When $\tau_c \geq 15$, all $L_\lambda^L(\theta)$ curves appear as a single line and reveal a monotonously decreasing pattern (limb darkening). The rate of limb darkening is independent to $r_c$ and $\theta_{oc}$. The rate is $5.5\% \pm 0.4\%$ for $\lambda = 500$ nm and $r_c = 1$ mm and is $27.9\% \pm 0.4\%$ for $\lambda = 862$ nm and $r_c = 1$ mm.

At SWIR ($\lambda = 2250$ nm), $L_\lambda^L(\theta)$ patterns also vary more with $\theta_{oc}$ for thin clouds ($\tau_c = 5$) than for thicker clouds [Figs. 8(c) and 8(d)]. Contrary to shorter wavelengths, the cloud droplet size ($r_c$) not only results in a variation of the $L_\lambda^L(\theta)$ pattern for thin clouds [Fig. 8(c)], but also causes a variation of the degree of limb darkening for moderate to thick clouds ($\tau_c = 15$) [Fig. 8(d)]. Clouds with a smaller $r_c$ ($4 \mu$m) appear less anisotropic than clouds with a larger $r_c$ ($14 \mu$m). The limb-darkening factor is $3.682 \pm 0.003$ for the former and is $4.517 \pm 0.028$ for the latter.

For snow grain size $r_s = 0.2$ mm, we do not present figures because the general patterns are similar to those for $r_s = 1.0$ mm. For $\tau_c \geq 15$, $r_c = 4 - 14 \mu$m and $\theta_{oc} = 30^\circ$, $65^\circ$, and $80^\circ$, the limb-darkening rates for $\lambda = 500$ and $862$ nm are $8.1\% \pm 0.3\%$ and $13.7\% \pm 0.3\%$, respectively. At $\lambda = 2250$ nm, the limb-darkening factor is $3.056 \pm 0.003$ for $r_c = 4 \mu$m and is $3.656 \pm 0.022$ for $r_c = 14 \mu$m. At $\lambda = 862$ and $2250$ nm, the difference in the limb-darkening rate of the $L_\lambda^L(\theta)$ patterns above snow packs with different snow grain sizes is caused by the difference in the snow surface BRF patterns that contribute to multiple scattering between snow and clouds. The stronger anisotropy of the snow surface BRF with larger snow grain size results in stronger anisotropy in the $L_\lambda^L(\theta)$ patterns.

In short, the $L_\lambda^L(\theta)$ patterns under thin clouds ($\tau_c = 5$) diverge significantly with the variation of $\theta_{oc}$ and exhibit the maximum at $\theta_s \sim \theta_{oc}$ for small $\theta_{oc}$ and at the middle range of $\theta_s$'s for large $\theta_{oc}$'s. As clouds become moderately thin ($\tau_c = 10$), the variation of $L_\lambda^L(\theta)$ with $\theta_{oc}$ decreases significantly. For moderate to thick stratus clouds ($\tau_c \geq 15$), the $L_\lambda^L(\theta)$ pattern has a stable limb-darkening pattern. At visible and NIR wavelengths, the rate of slant to moderate limb darkening ($\sim 8\%$ for $\lambda = 500$ nm and $14-28\%$ for $\lambda = 862$ nm) is independent of sky conditions, including variations of $\tau_c$, $r_c$, and $\theta_{oc}$, but dependent on $r_s$ for $\lambda = 862$ nm. At SWIR ($\lambda = 2250$ nm), the high rate of limb darkening (a factor of $3.06 - 4.52$) is independent of $\tau_c$ and $\theta_{oc}$, yet is dependent on both $r_c$ and $r_s$. For clouds between thin and moderately thick thicknesses ($\tau_c = 5-10$) under large $\theta_{oc}$'s ($\theta_{oc} = 65^\circ$ and $80^\circ$), the overall $L_\lambda^L(\theta)$ patterns appear less anisotropic than those under thicker clouds ($\tau_c \geq 15$), although the former is less stable and more sensitive to a slight change of sky conditions than the latter.

4. Patterns of Diffuse Sky Snow Surface Hemispherical-Directional Spectral Reflectance $R_\lambda^L(\theta)$

Using the same coupled cloud and snow model described in Subsection 3.A.2, we also simulated the patterns of the diffuse sky snow surface hemispherical-directional spectral reflectance $R_\lambda^L(\theta)$ under various sky and surface conditions. Figure 9 shows patterns of azimuthally averaged hemispherical-directional spectral reflectance as a function of viewing zenith angle $\theta_v$ for various sky conditions. The same patterns can also be viewed as the reciprocally derived direct beam spectral albedo estimates as a function of solar incidence angle $\theta_0$ with $\theta_0 = \theta_s$. Note that in each graph the $R_\lambda^L(\theta)$ values are given for all 72 sky conditions defined in Subsection 3.A.2. The dark blue and red curves provide mean values of $R_\lambda^L(\theta)$ for the cloud optical thickness ranges $\tau_c = 5-60$ and $\tau_c = 10-60$, respectively. The two curves are so close to each other that they form a single curve except for $\lambda = 415-610$ nm where the two curves slightly diverge only at large $\theta_v$'s [e.g., Fig. 9(a)]. The upper and lower boundaries of $R_\lambda^L(\theta)$ are also given by discrete symbols of the same colors for these two given $\tau_c$ ranges. It is obvious that the $R_\lambda^L(\theta)$ values are almost independent of $r_c$ and $\theta_{oc}$ for moderate to thick clouds ($\tau_c = 10-60$) although inclusion of thin clouds ($\tau_c = 5$) leads to much wider divergence because of the wide variation of the $L_\lambda^L(\theta)$ patterns with $\theta_{oc}$ above thin clouds (see Subsection 3.B.3). The simulated $R_\lambda^L(\theta)$ patterns at all the selected combinations of $\lambda$ and $r_c$ show a monotonous increasing trend with increasing $\theta_v$ [Figs. 9(a)–9(f)]. These results suggest that we can obtain consistent and stable estimates of the direct beam spectral albedo from the reciprocal approach for moderate to thick clouds ($\tau_c = 10-60$). In Figs. 9(a)–9(f) the directly simulated spectral albedo $\alpha_{\lambda,\theta}(\theta_s = \theta_v)$ are also given as continuous green curves for comparison. In addition, the cases with the minimum root-mean-square (rms) errors with respect to $\alpha_{\lambda,\theta}(\theta_0 = \theta_v)$ are presented as light blue curves in the graphs with the curve for Best.

4. Comparison and Error Patterns

From Figs. 9(a)–9(c) it is obvious that $R_\lambda^L(\theta)$ matches $\alpha_{\lambda,\theta}(\theta_0 = \theta_v)$ well for small to moderate $\theta_v$'s (or $\theta_0$'s) and then departs from $\alpha_{\lambda,\theta}(\theta_0 = \theta_v)$ more and more as $\theta_v$ further increases. To evaluate the overall performance of the reciprocal method, we plot in Fig. 10 the reciprocally derived $R_\lambda^L(\theta_v)$ against $\alpha_{\lambda,\theta}(\theta_0 = \theta_v)$ for all combinations of $16 \theta_0$'s, $6 \lambda$'s, $2 r_c$'s, $8 \tau_c$'s, $3 r_s$'s, and $3 \theta_v$'s listed in Table 1 except for the worst sky condition cases for small $\theta_{oc}$ ($30^\circ$) above thin clouds ($\tau_c = 5$). The diagonal line represents a one-to-one relation expected for the two sets of albedos. Large departures from the line mean a large error in the estimation. It is obvious that most points fall on or near the one-to-one diagonal line, indicating that the estimates from the reciprocal approach are generally good approximations of the direct beam spectral al-
The overall correlation coefficient \( r^2 = 0.9962 \). Note that there are still points that are far away from the one-to-one line. A close inspection of the data reveals that those points are cases for extremely large \( \theta_0 \)'s (86.1°, 88.4°, and 89.7°). When the points in the rectangles are omitted for those extremely large \( \theta_0 \)'s, the two sets of albedos match better with an improved overall correlation coefficient \( r^2 = 0.9996 \).

To evaluate the accuracy of the reciprocal method, we further look into the patterns of the error \( \Delta \alpha_{b,\lambda}(\theta_0 = \theta_0) \) for various wavelengths and snow grain sizes under various overcast sky conditions. Figure 11 is produced based on Eq. (8). The meanings of the symbols in Fig. 11 are the same as those in Fig. 9. When \( \theta_0 \)'s are not extremely large, the errors are small to moderate. More specific information about error patterns is given in Figs. 12 and 13, including rms errors in relation with sky conditions for the full \( \theta_0 \) range [Fig. 12(a)] and the reduced (\( \theta_0 < 83^\circ \)) range [Fig. 12(b)], enlargements of the errors for the solar zenith angle range (\( \theta_0 < 75^\circ \)) under typical cloud conditions (\( \tau_c = 10–60 \)) for medium [Fig. 13(a)] and large [Fig. 13(b)] snow grain sizes, and the max-
maximum absolute [Fig. 13(c)] and the maximum relative [Fig. 13(d)] errors for individual wavelengths. For visible and NIR wavelengths [Figs. 11(a)–11(d)], small positive errors are found for small to medium zenith angles ($\theta_0 \leq 45^\circ$) under typical overcast conditions ($\tau_c = 10$–60). The errors are positive because limb darkening in both $L^i_\lambda(\theta_i)$ [Figs. 8(a) and 8(b)] and $R^i_\lambda(\theta_0; \theta_i)$ [Figs. 6(a) and 6(c)] for small to medium $\theta_0$'s ($\leq 45^\circ$) results in positive covariance [see Eq. (11)]. For large and extremely large $\theta_0$'s, the errors become negative because the limb darkening of $L^i_\lambda(\theta_i)$ [Figs. 8(a) and 8(b)] and the limb brightening in snow surface $R^i_\lambda(\theta_0; \theta_i)$ [Figs. 6(b) and 6(c)] result in negative covariance [Eq. (11)]. The magnitude of the errors for large $\theta_0$'s ($60^\circ$–$68^\circ$) is moderate. For extremely large $\theta_0$'s, the magnitude of the errors increases rapidly with $\theta_0$. At medium zenith angles ($35^\circ \leq \theta_0 \leq 56^\circ$), the errors are minimum because of the transition of the error signs from positive to negative.

For SWIR ($\lambda = 2250$ nm), the errors are always negative because the snow surface $R^i_\lambda(\theta_0; \theta_i)$ patterns always show limb-brightening patterns [Figs. 6(a) and 6(d)] regardless of the solar incidence angle whereas $L^i_\lambda(\theta_i)$ exhibits a stable limb-darkening pattern. The increase of limb brightening of $R^i_\lambda(\theta_0; \theta_i)$ with $\theta_0$ leads to an increase in magnitude of the error with $\theta_0$ [Figs. 11(e), 11(f), 13(a)–13(d)]. At extremely large $\theta_0$'s, the errors are extremely large.

The best cases presented in Fig. 11 are the best sky conditions identified by their lowest rms error values among all sky conditions for individual $\lambda$–$r_s$ combinations and are denoted by encircled solid triangles in Fig. 12(a). Note that the best conditions are associated with thin clouds ($\tau_c = 5$) with large or extra large $\theta_0$'s. Looking back to Fig. 8, we find that those sky conditions have the maximum $L^i_\lambda(\theta_i)$ values within the middle range of $\theta_i$. When such a pattern is combined with either limb-darkening or limb-brightening patterns of the $R^i_\lambda(\theta_0; \theta_i)$ patterns, error values tend to be small to moderate based on Eq. (11). The signs of the individual errors depend on the signs of the covariance given in Eq. (11) and are not so straightforward as those for the typical cloud thickness range ($\tau_c = 10$–60).

Figures 12(a) and 12(b) both present rms of $\Delta\alpha_{b,\lambda}(\theta_0 = \theta_i)$ under various sky conditions. Out of 12 possible combinations of $\lambda$ and $r_s$, only five are selected. Because the patterns in the visible wavelength region ($\lambda = 415$–$665$ nm) are similar to each other, only a combination of $\lambda = 500$ nm and $r_s = 0.2$ mm is chosen. Similarly, many moderate to thick cloud optical thickness cases ($\tau_c = 15, 20, 30, 40$) were dropped, and only 36 sky conditions are selected from the 72 experimented sky conditions so that Figs. 12(a) and (b) not too busy. The symbols under the horizontal axis are organized in a $\theta_0$–$r_s$–$\tau_c$ interleaving manner to reflect variations of $\theta_0$, ($S$ for $30^\circ$, $L$ for $65^\circ$, and $X$ for $80^\circ$), $r_s$ (4, 8, 14 $\mu$m), and $\tau_c$ (5, 10, 25, and 60). The difference between Figs. 12(a) and 12(b) is that the former provides patterns of the rms of $\Delta\alpha_{b,\lambda}(\theta_0 = \theta_i)$ for the full $\theta_0$ range, and the latter gives a pattern for the reduced range ($\theta_0 < 83^\circ$) with the errors for the three largest $\theta_0$'s dropped. Because $\Delta\alpha_{b,\lambda}(\theta_0 = \theta_i)$ values for the largest $\theta_0$'s are the largest (Fig. 11), omission of the largest errors leads to smaller rms for the reduced $\theta_0$ range [Fig. 12(b)]. The best sky conditions are identified by their lowest rms error values for individual $\lambda$–$r_s$ combinations. Among all $\tau_c$'s, the thin clouds almost always give both the best overall results with large or extra large $\theta_0$'s and the worst ones (the largest rms) with small $\theta_0$'s. As $\tau_c$ increases, the difference in rms errors among different $\theta_0$'s decreases rapidly. At visible and NIR wavelengths, the rms of $\Delta\alpha_{b,\lambda}(\theta_0 = \theta_i)$ becomes independent of sky conditions as $\tau_c > 10$. At SWIR ($\lambda = 2250$ nm), the rms of $\Delta\alpha_{b,\lambda}(\theta_0 = \theta_i)$ is more sensitive to $r_s$ than to $\tau_c$ for $\tau_c = 10$. This difference can be explained by the fact that $L^i_\lambda(\theta_i)$ patterns are independent of $r_s$ at shorter wavelengths whereas the anisotropy of the $L^i_\lambda(\theta_i)$ patterns at $\lambda = 2250$ nm are sensitive to $r_s$ (see Subsection 3.B.3). The stronger anisotropy in $L^i_\lambda(\theta_i)$ resulting from larger $r_s$ leads to larger rms errors based on Eq. (11).

For all combinations of visible wavelengths ($\lambda = 415, 500, 610$, and $665$ nm) and snow grain sizes and for all moderate to thick overcast conditions ($r_s = 4, 8, 14$ $\mu$m; $\theta_0c = 30^\circ, 65^\circ$, and $80^\circ$; and $\tau_c = 15, 20, 30, 40,$ and 60), the average of the rms of $\Delta\alpha_{b,\lambda}(\theta_0 = \theta_i)$ over the full $\theta_0$ range is 0.0099 with a standard deviation of 0.0011. When the reduced $\theta_0$ range ($\theta_0 < 83^\circ$) is considered, the average and standard deviation values are reduced to 0.0044 and 0.0005, respectively.

At NIR wavelength ($\lambda = 862$ nm), the rms errors are also sensitive to snow grain size [Figs. 12(a) and 12(b)]. Under moderate to thick overcast conditions ($\tau_c = 10$–60), the statistics of the rms of $\Delta\alpha_{b,\lambda}(\theta_0 = \theta_i)$ for the full $\theta_0$ range are $0.014 \pm 0.0039$ and $0.028 \pm 0.00045$ for $r_s = 0.2$ mm and $r_s = 1.0$ mm, respectively. The statistics for the reduced $\theta_0$ range ($\theta_0 < 83^\circ$) are reduced to $0.0066 \pm 0.00025$ and $0.013 \pm 0.00030$, respectively. Over both full and reduced $\theta_0$ ranges, the averages of the rms of $\Delta\alpha_{b,\lambda}(\theta_0 = \theta_i)$ for large snow grains ($r_s = 1.0$ mm) are twice as large as
those for medium snow grains ($r_s = 0.2$ mm). Meanwhile, the standard deviations are all much smaller than the averages and do not vary much with snow grain size. These statistics again indicate that the errors of $\Delta \alpha_{b,\lambda}(\theta_0 = \theta_v)$ at this wavelength are moderate. They are larger than those in the visible wavelengths. In addition, errors are more sensitive to snow grain size than sky conditions.

At SWIR ($\lambda = 2250$ nm), the corresponding statistics for the medium and large snow grain sizes are $0.078 \pm 0.0037$ and $0.049 \pm 0.0018$ for the full $\theta_0$ range and are $0.026 \pm 0.0015$ and $0.0087 \pm 0.00043$ for the reduced $\theta_0$ range, respectively.

Figures 13(a) and 13(b) provide details of the patterns of $\Delta \alpha_{b,\lambda}(\theta_0 = \theta_v)$ under the typical cloud conditions ($\tau_c = 10–60$, $r_c = 4–14$ m, $\theta_0 = 30^\circ$, $65^\circ$, and $80^\circ$) as a function of $\theta_0$ at individual wavelengths for two snow grain sizes ($r_s = 0.2$ and 1.0 mm). The curves are successively displaced downward by an amount of 0.02 except for $\lambda = 415$ nm. Over the typical cloud optical thickness range ($\tau_c = 10–60$, the mean, maximum, and minimum values of $\Delta \alpha_{b,\lambda}(\theta_0 = \theta_v)$ at individual $\theta_0$ 's are given as solid curves and the discrete symbols close to the curves. The best sky conditions for individual $\lambda$–$r_s$ combinations for the full $\theta_0$ range are also provided. Figures 13(c) and 13(d) present the maximum deviations and the maximum relative errors in the
\(\alpha_{b,\lambda}(\theta_0 = \theta_c)\) for both snow conditions \((r_s = 0.2\) and \(1.0\) mm). The spectral curves are successively shifted by an amount of 0.01% and 1%, respectively.

It is obvious that under the typical cloud conditions the maximum deviations of reciprocity estimated \(\alpha_{b,\lambda}(\theta_0 = \theta_c)\) at visible and NIR wavelengths \((\lambda = 415–862\) nm) are within \(\pm 0.0035\) over the range \(\theta_0 < 57^\circ\). The corresponding relative errors are within \(\pm 0.5\%\). The errors become larger as \(\theta_0\) increases. For visible wavelengths \((\lambda = 415–665\) nm), the errors are still within \(\pm 0.008\) (\(\pm 0.9\%\)) up to \(\theta_0 = 74^\circ\). For the NIR wavelength \((\lambda = 862\) nm), the same accuracy can be achieved up to \(\theta_0 \sim 63^\circ\). The maximum deviation reaches \(-0.017\) when \(\theta_0\) approaches \(74^\circ\) and the snow grain size is large, although the maximum deviation is smaller (within \(\pm 0.009\%\) and 1%) for the smaller snow grain size \((r_s = 0.2\) mm) [Fig. 13(a)]. The albedo estimates at the SWIR wavelength \((\lambda = 2250\) nm) are accurate within \(\pm 0.009\) over the range \(\theta_0 < 57^\circ\) with the maximum deviations occurring with medium snow grain size \((r_s = 0.2\) mm). But the maximum relative errors (from \(-5\%\) to \(-13\%\)) for the same \(\theta_0\) range are associated with large snow grain size \((r_s = 1.0\) mm) because of the even smaller \(\alpha_{b,\lambda}(\theta_0 = \theta_c)\) values for the larger snow grains. As \(\theta_0\) further increases, the errors become larger.

5. Discussion and Conclusions

The spherical snow particle model is selected in our RT simulation based on our field observations of summer snow cover on sea ice in the Southern Ocean. It is well known that the spherical particle model results in the largest asymmetry factor in single scattering. Our trial simulations suggest that we can obtain smaller albedo errors with the reciprocal approach using smaller asymmetry factor values while keeping the single-scattering albedo unchanged in various cases. Therefore the assessment based on the spherical snow particle model tends to set an upper boundary of the albedo errors that can be achieved by the reciprocal approach for actual snow cover.

The simulated snow surface reflectance is strongly anisotropic when the solar zenith angle is large. The anisotropy is even stronger in the SWIR spectral region because stronger absorption reduces multiple scattering of solar radiation in the snow pack.

Under typical stratus clouds \((\tau_c = 10–60)\) the sky diffuse light over snow surface exhibits slight limb darkening at visible wavelengths, but shows a strong limb darkening at the SWIR. The anisotropy of sky diffuse light is independent of sky conditions, including variations of \(\tau_c, r_s,\) and \(\theta_0\), for all the wavelengths investigated except for \(\lambda = 2250\) nm where sky diffuse light slightly depends on \(r_c\).

Consequently, stable estimates of the snow surface direct beam spectral albedo can be derived by the reciprocal approach over a wide range of overcast sky conditions. The results are accurate to within \(\pm 0.008\) up to a solar zenith angle \((\theta_0)\) around \(74^\circ\) at visible wavelengths. The same accuracy can be achieved up to \(\theta_0 \sim 63^\circ\) for the NIR wavelength. The albedo estimates at \(\lambda = 2250\) nm are within \(\pm 0.009\) over the range \(\theta_0 < 57^\circ\), although the relative errors are large (5–13%). As \(\theta_0\) further increases, the errors become larger. Overall better estimates can be obtained from thin clouds \((\tau_c = 5)\) when \(\theta_0\) is large (65° and 80°). However, the estimates can become worse than those from the thicker cloud cases when \(\theta_0\) is small.

The new method helps expand the database of snow surface albedo for the polar regions where direct measurement of clear-sky surface albedo is lim-

**Fig. 12.** rms of errors under selected sky conditions for (a) the full \(\theta_0\) range and (b) reduced \(\theta_0\) range \((\theta_0 < 83^\circ)\). L, large; S, small; X, extra large; WV, wavelength; \(r_s\) snow grain radius.

**Fig. 13.** Details and statistics of \(\Delta\alpha_b(\theta_0 = \theta_c)\) patterns for \(\tau_c \geq 10\) for (a) \(r_s = 0.2\) mm, (b) \(r_s = 1.0\) mm, (c) maximum deviation for both \(r_s\), (d) maximum relative deviation for both \(r_s\). Wavelength is given as a parameter.
ited to large $\theta_0$’s only. The enhancement will assist
us in validating snow surface albedo models and
improving the representation of polar surface albedo in
GCMs.

Because the theoretical derivation is general for
any surfaces, the method is likely to be used at visible
and NIR wavelengths for any type of Earth surface
where sky is not obscured by terrain, trees, and build-
ings.

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References

1. J. E. Walsh, J. Curry, M. Fahnestock, M. C. Kennicutt II, A. D.
   McGuire, W. B. Rossow, M. Steele, C. J. Vorosmarty, and R.
   Wharton, Enhancing NASA’s Contribution to Polar Science
   cover monitoring: an update,” Bull. Am. Meteorol. Soc. 74,
3. H. J. Zwally, J. C. Comiso, C. L. Parkinson, W. J. Campbell,
   459 (National Aeronautics and Space Administration, Wash-
   ington, D.C., 1983).
4. C. L. Parkinson, J. C. Comiso, H. J. Zwally, D. J. Cavalieri, P.
   Gloersen, and W. J. Campbell, “Arctic sea ice 1973–1976 from
   489 (National Aeronautics and Space Administration, Wash-
   ington, D.C., 1987).
5. P. Gloersen, W. Campbell, D. J. Cavalieri, J. C. Comiso, C. L.
   Parkinson, and H. J. Zwally, “Arctic and Antarctic sea ice,
   1978–1987: satellite passive-microwave observations and analysis,”
   NASA Spec. Pub. 511 (National Aeronautics and Space Adminis-
6. J. Croll, Climate and Time in Geologic Relations: A Theory of
   Secular Change of the Earth’s Climate (Ibister, London, 1875).
   model to an increase of CO$_2$ concentration in the atmosphere,”
   CO$_2$ sensitivity experiments: snow-sea ice albedo parameter-
   izations and globally averaged surface air tempera-
   albedo feedback in a CO$_2$-doubling simulation,” Clim. Change
    Vetterling, Numerical Recipes in C: The Art of Scientific Com-
    Zakharova, “Bidirectional reflectance of flat, optically thick
    particulate layers: an efficient radiative transfer solution
    and applications to snow and soil surfaces,” J. Quant. Spec-
12. X. Zhou, “Optical remote sensing of snow on sea ice: ground
    measurements, satellite data analysis, and radiative transfer
    modeling,” Ph.D. dissertation (University of Alaska, Fair-
    banks, Alaska, 2002).
13. T. H. Painter and J. Dozier, “Measurements of the bidirec-
    tional reflectance of snow at fine spectral and angular reso-
    lution,” in Proceedings of the 70th Annual Western Snow
    Conference, available online at http://www.westernsnow
14. K. Morris and M. O. Jeffries, “Seasonal contrasts in snow cover
    characteristics on Ross Sea ice floes,” Ann. Glaciol. 33, 61–68
42. S. Li, “A model for the anisotropic reflectance of pure snow,” M.A. thesis (University of California at Santa Barbara, Santa Barbara, Calif., 1982).